1) When $\Lambda = 0$ we find, of course, that $U = U_0$; also $b/b_0 = [1 + 4k\tilde{x}]^{1/2}$ and $u_c/u_{c0} = [1 + 4k\tilde{x}]^{-1/2}$ for the plane case; and $b/b_0 = [1 + 6k\tilde{x}]^{1/3}$ and $u_c/u_{c0} = [1 + 6k\tilde{x}]^{-2/3}$ for the axisymmetric case.

2) As in the laminar flows, there exist exponential solutions but in the axisymmetric case as well as in the plane case. In the plane case, we find that when $\Lambda = -4k$, then $U/U_0 = \exp(-4k\tilde{x})$, $b/b_0 = \exp(6k\tilde{x})$, and $u_c/u_{c0} = \exp(2k\tilde{x})$. For the axisymmetric case we also find that when $\Lambda = -4k$, then $U/U_0 = \exp(-4k\tilde{x})$, but $b/b_0 = \exp(4k\tilde{x})$ and $u_c/u_{c0} = 1$.

3) The case where u_c/U = const occurs when $\Lambda = -k$ in the plane case and $\Lambda = -2k$ in the axisymmetric case.

Hill[6] has presented some experiments and theory on turbulent plane wakes in pressure gradients. One of his experiments corresponds rather closely to the condition u_c/U = const = 0.5, and we find that $\Lambda \simeq -k = -0.078$. Thus, as Hill comments, the value k seems to be independent of the pressure gradient; this is analogous to Clauser's discovery[7] in the case of "equilibrium," turbulent boundary layers.

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An Energy Inventory in a Coaxial Plasma Accelerator Driven by a Pulse Line Energy Source

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Introduction

THE most severe limitation of the many plasma accelerators that have been considered for space-propulsion applications is low electrical efficiency.[1] In order to improve the efficiency of a specific accelerator it is necessary to know how the source energy is partitioned within the accelerator at any time.

In the experiments described in this paper a pulsed coaxial plasma gun, powered by a pulse-line energy source, was studied. The partition of energy among the source, the magnetic field, and the work done on the plasma was determined from measurements of the magnetic field within the gun, together with the voltage and current at the terminals of the gun. A pulse line energy source[2] was used because it can deliver a constant current at a constant voltage for most of its period.

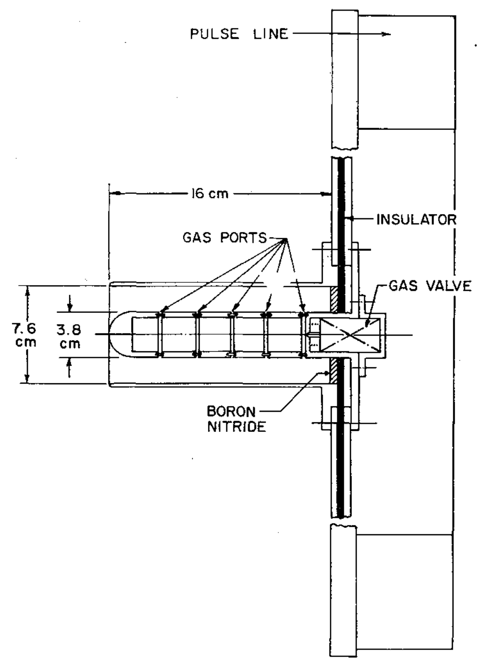

Fig. 1 Schematic diagram of the accelerator.

Description of the Apparatus

The plasma accelerator[2] is illustrated schematically in Fig. 1. The energy storage capacitor is a single unit, wound in the form of a toroid with an outside diameter of 22 in., an inside diameter of 19 in., and 12 in. in length. This capacitor exhibits a pulse-line behavior because the wave propagation time in the capacitor is comparable to the period of the system.[2] The line impedance is approximately 17 $m\Omega$, the pulse time 0.8 μsec, and the total capacitance 22 μF. The unit is operated at 6.3 kv with nitrogen as propellant. The propellant is injected through a solenoidal gas valve and is dispensed through multiple gas ports into the

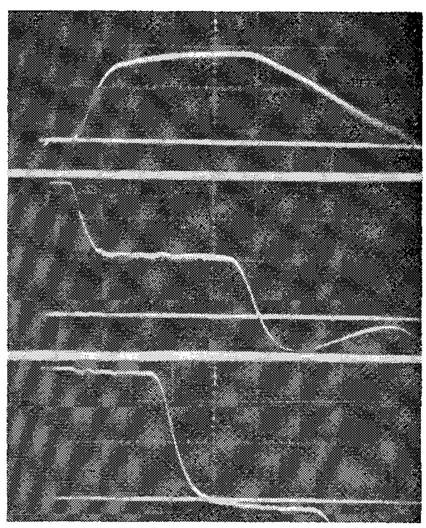

Fig. 2 Current and voltage waveforms for the pulse line: $I \sim 10^5$ amp/cm, $V \sim 2$kv/cm, $t = 0.2\ \mu$ sec/cm.

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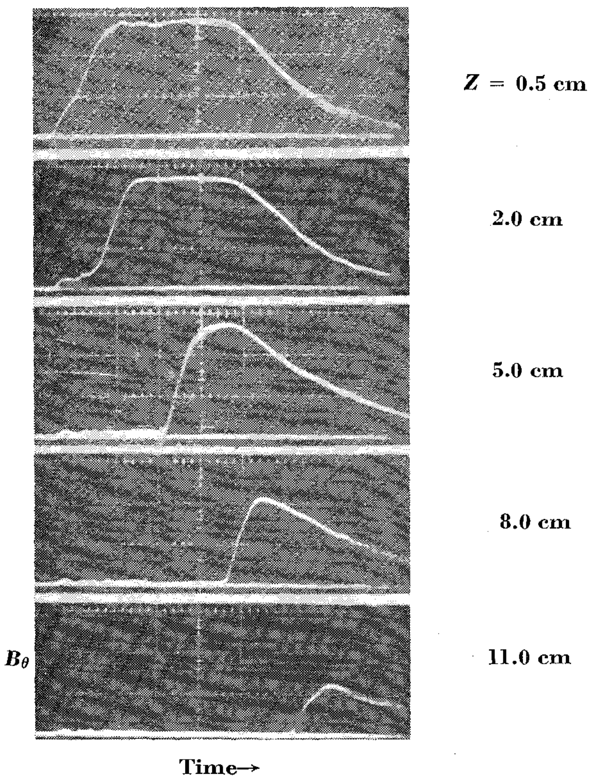

Fig. 3 Oscillograms of B_θ: Z = axial position, B_θ = 5.4 kgauss/cm, t = 0.2 μsec/cm.

annular region between the electrodes. The procedure is to charge the capacitor, pulse the gas valve, and wait until pressure in the gun rises to the level appropriate to breakdown. The breakdown occurs near the boron nitride insulator at the breech of the gun.

Experimental Results

Oscillograms of the current to the gun, the voltage on the gun, and the voltage at the open end of the pulse line are shown in Fig. 2. The current rises to about 200 ka in 0.15 μsec and then remains essentially constant for 0.7 μsec; the voltage at the gun drops to slightly less than one half of the initial voltage and then swings negative at the time of arrival

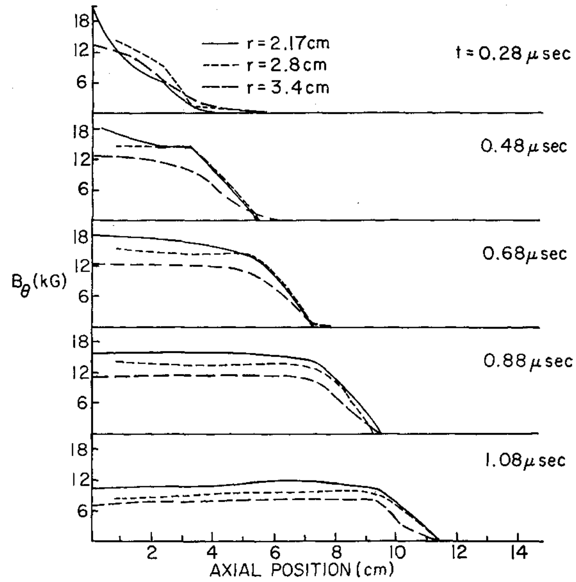

Fig. 4 Axial distributions of B_θ, r = radius.

of the voltage wave reflected from the open end of the line; the voltage at the open end of the line is unchanged until the primary voltage wave arrives and reduces it to near zero. The waveforms demonstrate that after 0.15 μsec the impedance of the gun is very nearly constant at 14 $m\Omega$.

The current sheet maintains azimuthal symmetry, and the only significant component of magnetic field is B_θ; this was measured at three radii and at 1 cm intervals along the barrel. The B_θ oscillograms in Fig. 3 were taken at midradius; each photograph shows four successive traces overlaid. In Fig. 4 the axial distribution of B_θ at various times is plotted for the three radial positions. A clearly defined current sheet forms in a few tenths of a microsecond, then travels along the barrels at a velocity of 10 cm/μsec.

Because $\partial B_\theta/\partial z = 0$ behind the current sheet, it is apparent that no current flows in this region; because $\partial B_\theta/\partial t$ is also small, the radial electric field can then be determined from the terminal voltage and the electrode separation. Within experimental error this electric field is equal to the product of the measured sheet velocity and the measured B_θ. Therefore, the current sheet is moving at the same velocity as the flux lines and, consequently, if there is any plasma behind the sheet, it must also be moving with this velocity. The gas encountered by the moving current sheet must be either swept along, or driven into the electrodes, or both. A few neutral particles may be left in the wake but, unless their density is much lower than the original gas filling, they too must be traveling close to the sheet speed or they will experience a radial electric field sufficient to cause breakdown, and radial currents behind the main current sheet would result.

From the data presented numerical values can be determined for lumped circuit parameters which describe the electrical behavior of the gun. It is permissible to talk of an inductance L and its time derivative $\dot{L}$ if the profile of the current sheet is steady and if the magnetic energy within the sheet is small compared with the total energy in the system. The integral of the power delivered to the gun is[3]

$$\int_0^t P\,dt = \frac{1}{2}LI^2 + \frac{1}{2}\int_0^t I^2\dot{L}\,dt + \int_0^t I^2 R$$

The first term on the right side is the magnetic energy inside the gun; this term finally must go to zero. The second term equals the work done on the plasma which goes into kinetic energy, internal energy, and losses. The last term is the energy dissipated in ohmic losses and is much smaller than the other two terms. All the terms in the equation can be calculated from the measurements described; Fig. 5 shows how they vary with time. Of the energy stored in the pulse line, 88% has been transferred to the gun at 0.8 μsec when voltage reversal occurs; up to this time the energy is shared approximately equally between the magnetic field and the work done on the current sheet. After voltage reversal some of the magnetic energy in the barrel is withdrawn and returns to the pulse line, while the rest continues to do work on the current sheet. The total energy supplied to the gun is 69%, of which 58% can be identified as work done on the plasma. The difference is presumably due to resistive losses and experimental error.

Discussion and Conclusions

The results presented show that a large fraction of the electrical energy stored in the pulse line is delivered to the plasma. However, for efficient plasma acceleration, it is essential that most of this energy be in directed kinetic energy when the plasma leaves the gun. Measurements of the neutral gas density show that at the moment of breakdown the barrels are uniformly filled with gas up to the muzzle. Because the current sheet is moving at a constant velocity, the energy supplied to the plasma partitions approximately equally between the directed kinetic energy and the various

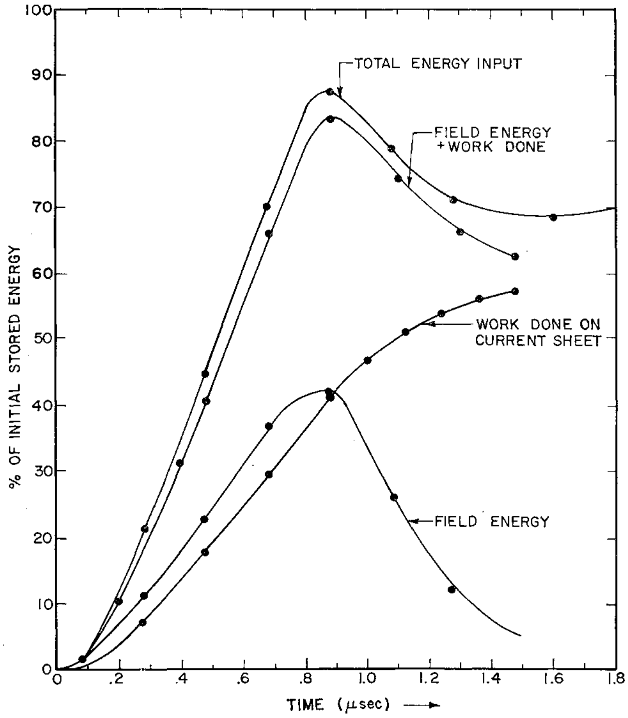

Fig. 5 Energy inventory for the accelerator.

forms of internal energy.[3] Some of the energy in thermal motion of the ions and electrons may subsequently appear in directed energy as a result of expansion near the muzzle of the gun. However, only a small amount of thermal energy can possibly be reclaimed for the following reasons. The plasma density is about 10^{16} cm^{-3} and, in a plasma of high Z material like nitrogen, the electron temperature will be radiation-limited to about 5–10 ev[2,4,5]; consequently the thermal energy stored in the electrons is small compared with the average ion energy in directed motion ($\approx$700 ev). The ions will rapidly lose their thermal energy by coulomb collisions with the cold electrons and will have little left when they reach the muzzle (the e folding time is $\approx$0.25 μsec for 5 ev electrons at a density of 10^{16} cm^{-3}).[6]

From the preceding arguments about 30% of the initial stored energy is expected to appear in the exhaust. (Ionization energy is insignificant compared with the directed energy.) A calorimeter in the exhaust collected only 15% of the initial stored energy; in addition, measurements in the exhaust plasma[7] showed that the average velocity was 6 cm/μsec compared with the sheet speed of 10 cm/μsec. When the barrel length was reduced from 16 to 9 cm, so that the current sheet reached the muzzle at the time of voltage reversal, the calorimetric efficiency increased to 30% and the average velocity of the exhaust plasma equaled the sheet speed. These two results suggest that increased wall losses occur if the current sheet is still inside the barrels when the current decays.

By improving the match between the load and the energy source and using larger diameter barrels, efficiences up to 45% have been obtained, but it is difficult to see how an over-all efficiency greater than 50% can be achieved if most of the mass accelerated is picked up by the current sheet while it is traveling at its terminal velocity. Higher efficiencies are theoretically possible if a plasma slug of constant mass is accelerated.[3,8,9] However experiment has indicated that the current sheet in a coaxial gun tends to become unstable unless it continually sweeps up gas inside the gun.[10] Such an instability may make the simple slug model impossible to achieve. With a gas-filled gun the slug model may be appropriate after the plasma reaches the muzzle if appreciable acceleration takes place outside the gun where the neutral gas density is very low; in this case the efficiency could exceed 50%.

An alternative approach to the problem of improving efficiency in the pulsed coaxial rail accelerator is to continually feed propellant into the gun at the breech. If propellant is fed in at a sufficient rate then the current sheet should remain stationary while plasma accelerates through it. This is a deflagration-type phenomenon and high efficiencies can be expected; this expectation is encouraged by the performance of thermo-ionic arc jets.[1]

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A Solution of the Base Pressure Problem Applicable to Transient External Flows

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WAKE flows have been of interest for a considerable time and both steady-flow cases and quasi-steady aspects of transient cases have been investigated.[1–4] All of these investigations related the dynamics of the external isentropic stream and the dissipative mechanism of the jet mixing region.

Inspection of the system of equations describing the mechanism of wake flow under steady condition[2] does not give much encouragement for extending it formally to include all possible additional nonsteady terms. A quasi-steady solution has been made that shows good agreement with limited experimental checks.[4] This quasi-steady theory contains several limitations that restrict its application to wake flows involving time-dependent, external stream properties.

Referring to a simple backstep geometry (see Fig. 1), the quasi-steady theory presented previously[4] assumes that the transient properties of the approaching stream number M_{1a} (or Crocco number C_{1a}), stagnation pressure P_{01a}, and the stagnation temperature T_{01a} are varying slowly enough such that the volume of the wake is not materially effected (θ_{2a} remains substantially constant). For the same reason it is

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